ELASTO-PLASTIC ANALYSIS OF BURIED PIPELINES SUBJECTED TO GROUND DEFORMATION

by

Hiroshi Sato and Nobutaka Ishikawa,CSCE

Department of Civil Engineering
National Defense Academy,Japan

ABSTRACT

This study presents a quasi-static approach using a matrix method based on the linear programming(LP) for the elasto-plastic analysis of a buried pipeline subjected to ground deformation. The actual buried pipeline is initially modeled as a discrete pipeline-ground system,in which pipe and ground spring elements are supposed to have elastic-perfectly plastic behaviors,respectively. The analysis procedure involves a performance of a series of incremental deformation analyses as LP problem. Herein,the objective is to maximize ground deformation increment during the successive yielding of ground spring or pipe bar element, while simultaneously satisfying the conditions of compatibility, equilibrium,yield and plastic flow as constraints. Three examples are analyzed to illustrate the features and scope of application of the approach.

INTRODUCTION

According to the records of past earthquake damage,most of seismic damage of buried pipelines is mainly caused by the permanent ground deformation such as relative ground movement,fault motion and travelling seismic waves. Therefore,as a first step to simulate the seismic behavior of buried pipelines,it is necessary to develop the elasto-plastic analysis method of the buried pipelines subjected to ground deformations. As for the work of this area,several methods (1～4) based on the seismic deformation method have been proposed in a variety of ways. However,these methods (1～4) can not exactly investigate the elasto-plastic behavior of buried pipeline,since they can not treat the range beyond the elastic limit of pipe. On the other hand,there is a tendency to utilize the ductility of steel pipe beyond the elastic limit,due to the recent rapid progress of pipe material and the accumulation of experimental data. In fact,a new design method (5) has been proposed to permit the 1 % strain of pipe as the design criterion for the high pressure gas pipeline in Japan. This indicates a new and revolutionary direction for the earthquake resistant design of buried pipeline. In the analytical field,however,the elasto-plastic analysis method has not yet been established for the case including the plastic strain of pipe.

To overcome these difficulties mentioned above, this paper presents a new seismic deformation method using linear programming(LP) for the incremental analysis of buried pipeline subjected to the ground deformations. The approach essentially extends the elastic-plastic analysis of above ground structures(6) and the beam on the foundation(7) using LP. Here, the analysis process involves the maximization of ground displacement increment as an objective criterion, while simultaneously satisfying the conditions of compatibility, equilibrium, yield and plastic flow as constraints. Upon succeeding iterations, the final solutions are achieved when the ground displacement factor increment becomes zero.

In numerical examples, the buried pipelines with or without joints subjected to axial ground deformation are initially presented to illustrate the features of the approach. Then, the axial and flexural behaviors beyond elastic limit of pipe are investigated for the various ground deformations.

ELASTO-PLASTIC ANALYSIS OF BURIED PIPELINES BY LINEAR PROGRAMMING

Herein, the basic equations will be explained for the axial behavior of a buried pipeline and they will be also applied for the flexural behavior in the numerical example. In addition to the usual assumptions of the seismic deformation method and simple plastic theory, it is assumed that an actual pipeline is modeled as the discrete structure as shown in Fig.1; the pipeline is characterized by a series of "bar" elements, and the ground is characterized by a series of "spring" elements. The elastic-perfectly plastic behavior characteristics are assumed for both elements as shown in Fig.2(a),(b).

Using the external ground displacement instead of the external load in the formulation of beam on the foundation(7), the basic conditions for the elasto-plastic analysis of the buried pipeline are obtained as the following incremental forms, i.e., Eqs.(1b)~(1ℓ). As these relations are established for any ground displacement increment $\Delta\alpha (\leqq \Delta\alpha_P$: the ground displacement factor increment from the yielding of one element to the another), they can pursue the successive yielding of elements by maximizing $\Delta\alpha$. Consequently, the elasto-plastic incremental analysis of a buried pipeline subjected to the ground displacement may be formulated as the following LP problem:

Given : C_N , C_S , L_N , L_S , N_P , S_P , k_N^{-1}, k_S^{-1}, u_S , $\bar{N}$, $\bar{S}$

Find : ΔN , ΔS , Δu, $\Delta\lambda_N$, $\Delta\lambda_S$, Φ_N , Φ_S

Such that : $\Delta\alpha$ --→ maximize (1 a)

Subject to :

Ⓔ $C_N^T \Delta N + C_S^T \Delta S = 0$ (1 b)

$$Ⓒ\quad L_N\,\Delta\lambda_N + K_N^{-1}\,\Delta N - C_N\,\Delta u = 0 \qquad (1\ c)$$

$$L_S\,\Delta\lambda_S + K_S^{-1}\,\Delta S - C_S(\Delta u - \Delta\alpha\, u_S) = 0 \qquad (1\ d)$$

$$Ⓨ\quad L_N\,\Delta N + (-\Phi_N) = L_N^T\,N - N_P\ ,\ -\Phi_N \geqq 0 \qquad (1\ e)$$

$$L_S\,\Delta S + (-\Phi_S) = L_S^T\,S - S_P\ ,\ -\Phi_S \geqq 0 \qquad (1\ f)$$

$$Ⓟ\quad (\,L_N^T\,N - N_P)^T\Delta\lambda_N = 0,\ (\,L_S^T\,S - S_P)^T\Delta\lambda_S = 0 \qquad (1\ g),(1\ h)$$

$$(\,L_N^T\,\Delta N\,)^T\Delta\lambda_N = 0,\ (\,L_S^T\,\Delta S\,)^T\Delta\lambda_S = 0 \qquad (1\ i),(1\ j)$$

$$\Delta\lambda_N \geqq 0\ ,\ \Delta\lambda_S \geqq 0 \qquad (1\ k),(1\ \ell)$$

in which Eq.(1a) means the activiation of a new yield mode. Eq.(1b) expresses the equilibrium condition,Eqs.(1c),(1d) define the compatibility equations for the pipe and the spring elements,respectively. Eqs.(1e), (1f) are the yield conditions for the pipe and the spring elements,respectively,and Eqs.(1g),(1i) and Eqs.(1h),(1j) are the plastic flow conditions of the pipe and the spring elements,respectively. It is noted that Eqs.(1i),(1j) consider the unloading phenomenon,i.e.,they ensure that plastic flow can not occur ($\Delta\lambda_N^i = 0$) if the stress increment decreases ($\Delta N_i < 0$). Abbreviations Ⓔ,Ⓒ,Ⓨ,Ⓟ mean the conditions of equilibrium,compatibility,yield and plastic flow,respectively.

In Eq.(1),the increments of stress and deformation are simultaneously found by only a LP application and,as such,they are referred as a modified forced method. Having the increments $\Delta\alpha$, ΔN, ΔS, Δu, $\Delta\lambda_N$, $\Delta\lambda_S$ from Eq.(1),the total quantities in the present stage are found by adding the increments to the values in the previous stage. The specially devised computer program determines the stresses and deformations at each stage as well as the sequence of the yielding elements. That is, the computer program initially involves the determination of the first yield element with the ground displacement factor α_1 in which the maximum stress has just reached the elastic limit. Then,the second yield element with α_2 is found by using Eq.(1) with the previous data. Upon succeeding iterations,if the analysis finds $\Delta\alpha = 0$,the operation is completed.

EXAMPLES

EXAMPLE 1 : In order to illustrate the features of the present method, the axial behavior is firstly investigated for a straight steel pipeline (ℓ =100 m) with or without expansion joints. The pipeline is discretized into the 20 pipe elements as shown in Fig.1 and properties of pipe and ground soil are summarized in Table 1 and Table 2. Note that both end elements ①, ㉒ in Fig.1 are selected as dummy elements in order to express the joints elements or the actual fixed conditions. For example,adopting the small rigidity at both end elements,say,1/1000 of other pipe elements,this model is transformed into the pipeline with expansion joints. The two types of sinusoidal waves are chosen as the input axial ground displacements u_s ,i.e.,the one wave and the half wave length as shown in Fig.3.

Performing incremental analysis in case of pipeline with expansion joint,the historical collapse modes with α_{max} for the two types of ground displacements are found to be shown in Fig.3. Note that the numerals of yielded elements in Fig.3 indicate the sequence of yielding. It should be noted from Fig.3 that the final collapse modes in both cases are completed by only yielding of ground spring elements. The maximum ground displacement factor α_{max} and the maximum strain of the pipeline ε_{max} are summarized in Table 3 for the various combinations of the ground displacement mode,the existence of joint and the limit relative displacement e_y .

Figures 4 and 5 show the relationship between the ground displacement factor α and the maximum strain of pipe ε_{max} in both cases of ground displacement of one wave length and half wave length,respectively. It is found from Fig.4 that the maximum strain ε_{max} with joints becomes larger than the one without joints. On the other hand,it should be noted from Fig.5 that ε_{max} with joints decreases about 1/2 in comparison with the one without joints.

EXAMPLE 2 : The pipeline subjected to the forced axial displacement at both ends as shown in Fig.6(a) is analyzed to investigate the elasto-plastic behavior of pipeline in the case occurring plastic strain of pipe. From incremental analysis,the historical collapse mode and the displacement factor-strain curves are found to be shown in Figs.6(b) and 7,respectively.

(a) It is found from Fig.6(b) that the final collapse mode of pipeline-ground system is achieved by only yielding of pipe elements.

(b) It should be noted,however,that unloading phenomenon occurs for the ground spring elements in the yielding process even when the applied ground displacement increases. Consequently,the frictional resistant forces of these elements are equal to zero.

(c) It is recognized from Fig.7 that the ground displacement factor becomes α = 22 corresponding to the compressive yield strain ε_{cr} (= 35 t/(D-t) = 0.69 %),which is specified in the " Recommended Practice for Earthquake Registant Design of High Pressure Gas Pipeline "(5).

EXAMPLE 3 : This approach is applied to the flexural behavior of the buried pipeline subjected to the ground displacement perpendicular to the pipeline axis. It is assumed that the ground reaction coefficient in perpendicular direction to the pipeline axis is constant (k_{sy} = 76.6 kg/cm^2) and the ground spring force is also constant ($R_p = k_{sy} \cdot a_o \cdot e_y$ = 38.3 ton) throughout. Figures 8(a),(b),(c) show the input ground displacement mode,the output historical collapse mode and the deflection curves of pipe,respectively. It is found from Figs.8(b) and (c) that the final collapse mode is formed with only yielding of pipe and,as such,the earthquake resistant capacity for the flexural deformation is remarkably larger than that for the axial deformation.

CONCLUSION

The following conclusions may be drawn from this study.

(1) An efficient analysis method for the elasto-plastic behavior of buried pipeline has been developed by extending the technique for the above ground structure.

(2) This approach can exactly evaluate the strain and the deflection of pipeline beyond the elastic limit of pipe.

(3) It is found from examples that there are two types of collapse modes,i.e.,the one is due to only yielding of ground springs as shown in Fig.3, and the other is due to only yielding of pipe elements as shown in Figs.6 and 8 .

(4) It should be noted that the maximum strain with joints is larger than that without joint for the ground deformation of one wave length. Therefore,the position of joint should be carefully arranged in the actual pipeline system.

(5) It is recognized that local unloading of a yield element occurs for the increasing ground deformation after a certain amount of plastic deformation has taken place. Such a phenomenon can only be detected by a complete incremental analysis.

(6) The present approach will be able to give us a useful information on the earthquake resistant capacity,if the properties of pipe and ground soil are known for the existing buried pipeline in a local area.

REFERENCES

(1) Sakurai,A.et al.,"Dynamic Stresses of Underground Pipe Lines during Earthquake 2",Tech. Rept. No.69087,Central Research Institute of Electric Power Industry (1970) (in Japanese).

(2) Kitade,K.et al.,"Aseismic Strength of Underground Pipeline",Tech. Rept. of Mitsubishi Heavy Industries,Vol.11,No.4(1974)(in Japanese).

(3) Takada,S.,"Seismic Response Analysis of Buried PVC and Ductile Iron Pipelines",Recent Advances in Lifeline Earthquake Engineering in Japan-PVP-43,ASME,(1979).

(4) Miyajima,N.et al.,"An Example of Seismic Design and Earthquake Response Measurement of Buried Pipeline",Proceedings of U.S.-Japan Seminar on Earthquake Engineering Research with Emphasis on Lifeline Systems,(1976).

(5) The Japan Gas Association,"Recommended Practice for Earthquake Resistant Design of High Pressure Gas Pipeline",(1982) (in Japanese).

(6) Ishikawa,N.,"Load-Deformation Analysis of Elastic-Plastic Frames by Linear Programming",Theoretical and Applied Mechanics,Vol.23, Univ. of Tokyo Press,(1975).

(7) Ishikawa,N.et al.,"Elastic-Plastic Analysis of Beams on Elastic-Plastic Foundation by Linear Programming",Memoirs of the National Defense Academy,Japan,Vol.22,No.4,(1982).

NOTATION

a_o	: length of pipe element
$\mathbf{C}_N$, $\mathbf{C}_S$	: compatibility matrix relating to pipe element,ground spring element
$e = e_e + e_p$	: relative displacement between pipe and ground
e_e	: elastic part of e
e_p	: plastic part of e
e_y	: limit relative displacement
$\mathbf{k}_N^{-1}$, $\mathbf{k}_S^{-1}$	: assemblage flexibility matrix relating to pipe element, ground spring element
$\mathbf{L}_N$, $\mathbf{L}_S$	: matrix whose raws are the outward unit normal vectors to the yield planes relating to N , S
$\mathbf{N}$, $\mathbf{S}$	: vector of axial force,frictional resistant force
$\Delta\mathbf{N}$, $\Delta\mathbf{S}$	: increment vector of $\mathbf{N}$, $\mathbf{S}$
$\mathbf{N}_p$, $\mathbf{S}_p$	: limit value of $\mathbf{N}$, S
$\mathbf{u}$	: vector of nodal displacement of pipe element
$\Delta\mathbf{u}$	: increment vector of $\mathbf{u}$
$\mathbf{u}_S$	: vector of ground displacement
α	: ground displacement factor
α_{max}	: maximum ground displacement factor
$\Delta\alpha$	: increment of α
$\delta = \delta_e + \delta_p$	: expansion or contradiction of pipe element
δ_e	: elastic part of δ
δ_p	: plastic part of δ
δ_y	: yielding value of δ
Φ_N , Φ_S	: vector of yield function of pipe and spring elements
$\Delta\lambda_N$, $\Delta\lambda_S$	: vector of plastic multipliers increment of pipe and spring elements

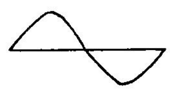
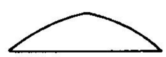
Table 1 Dimensions of pipe

outer diameter	D	40.64	cm
wall thickness	t	0.79	cm
cross-sectional area	A	98.9	cm^2
Young's modulus	E	2.1×10^6	kg/cm^2
moment of inertia	I	19640	cm^4
ultimate strength of pipe	σ_u	4100	kg/cm^2
yield stress of pipe	σ_y	2300	kg/cm^2
axial plastic capacity	$N_p(=\sigma_y \cdot A)$	227470	kg
plastic moment	$M_p(=\sigma_y \cdot I \cdot 2/D)$	2222×10^3	kg·cm
length of model pipeline	ℓ	100	m
length of pipe element	a_o	500	cm
axial stiffness of pipe element	$K_p(=E \cdot A/a_o)$	415380	kg/cm

Table 2 Properties of ground spring element

axial direction			
limit frictional spring force	$S_p(=K_{sx} \cdot e_y)$	5745 (1915)	kg
ground spring constant	$K_{sx}(=k_{sx} \cdot a_o/2)$	19150	kg/cm
ground spring constant per unit length	k_{sx}	76.6	kg/cm^2
limit relative displacement	e_y	0.3 (0.1)	cm
perpendicular direction			
limit ground reaction force	$R_p(=K_{sy} \cdot e_y)$	38300	kg
ground spring constant	$K_{sy}(=k_{sy} \cdot a_o)$	38300	kg/cm
ground spring constant per unit length	k_{sy}	76.6	kg/cm^2
limit relative displacement	e_y	1.0	cm

Table 3 Maximum ground displacement factor(α_{max}) and maximum strain(ε_{max}) (Example 1)

Ground displacement mode	Joint at both ends	Limit relative displacement (e_y)	0.1 cm	0.3 cm
	without	α_{max}	0.458	1.373
		ε_{max}	82	248
	with	α_{max}	1.519	4.572
		ε_{max}	156	468
	without	α_{max}	1.198	3.597
		ε_{max}	174	524
	with	α_{max}	1.329	4.099
		ε_{max}	84	250

(ε_{max} : $\times 10^{-6}$)

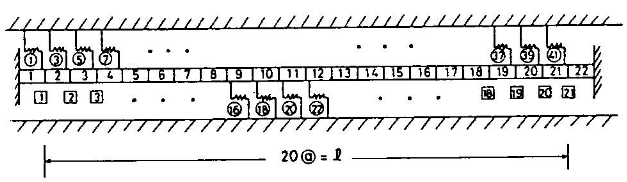


Fig.1 Discretized pipeline model

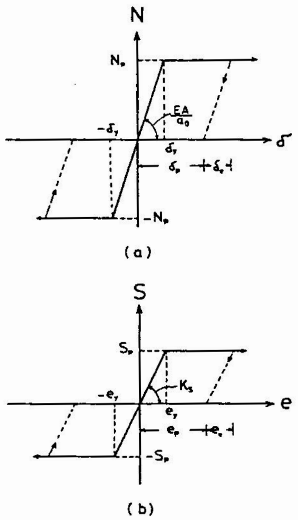


Fig. 2 Elastic-plastic behaviors of (a)pipe element and (b)ground spring

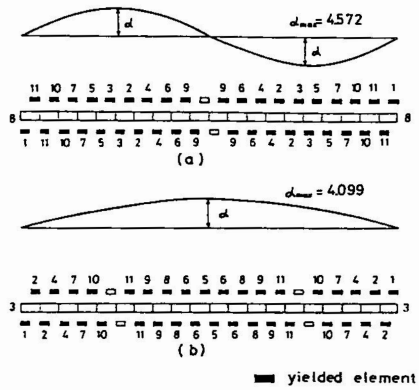


Fig. 3 Example 1: Behavior of pipeline with expansion joints at both ends(e_y=0.3cm) for the ground displacements of (a) one-wave length and (b) half-wave length

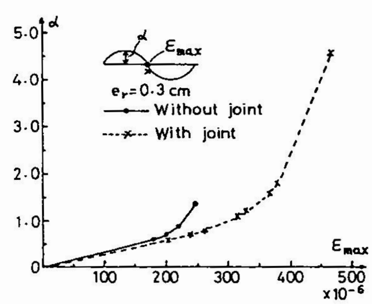


Fig.4 Example 1: Ground displacement factor-maximum strain curves with or without joint (one wave length)

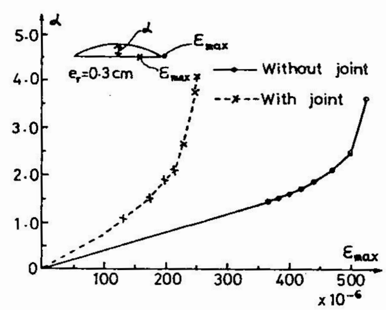


Fig.5 Example 1: Ground displacement factor-maximum strain curves with or without joint (half wave length)

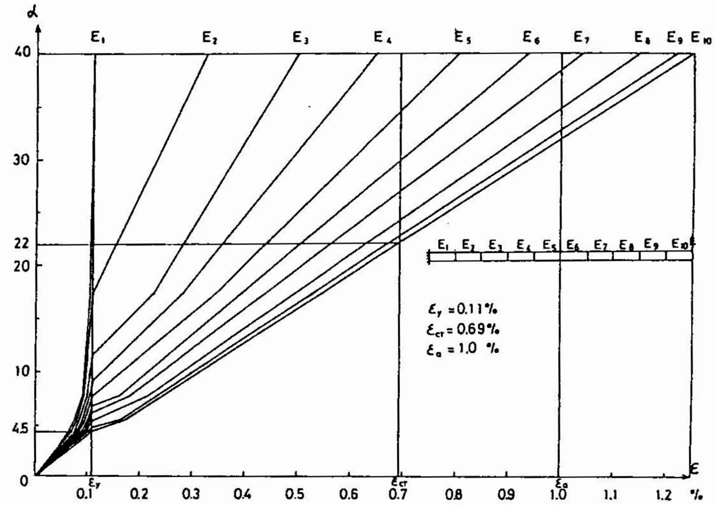


Fig. 7 Example 2 : Ground displacement factor-strain curves of pipe element

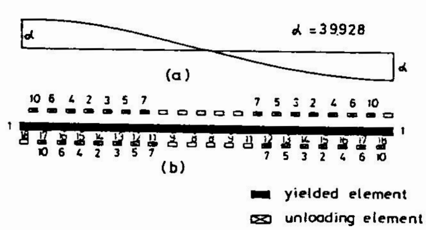


Fig.6 Example 2 : Behavior of pipeline subjected to forced displacement at both ends (e_y=0.3cm)

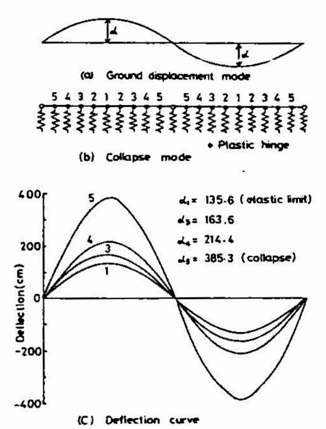


Fig.8 Example 3 : flexural behavior of pipeline